to constancy for months at a time, it may show large variations over a period of years. It is largely dependent on circumstances other than the geographical co-ordinates of the stations. For a complete elucidation of the phenomena much further investigation is required, in which account must be taken of all the magnetic elements.

In the present enquiry the author is much indebted to the kind co-operation of the Rev. A. L. Cortie, S.J., and Dr. Crichton Mitchell, F.R.S.E. The former lent a number of Stonyhurst curves, while the latter supplied the Eskdalemuir curves with full particulars of the hourly values and base-line values.

A more complete discussion, with full particulars of the observational data, will be published in the 'Geophysical Memoirs' of the Meteorological Office.

Siren Harmonics and a Pure Tone Siren.

By E. A. Milne and R. H. Fowler.

(Communicated by Prof. A. V. Hill, F.R.S. Received November 9, 1920.)

§ 1. *Introduction.*—This paper deals with the purity of tone of the sound produced by an ordinary Seebeck siren, and with the problem of designing a siren to produce a pure, or comparatively pure, tone. The results were obtained in the course of certain technical experiments on the location of aircraft by sound; they seem, however, to be of more general interest, as a source of sound giving a fairly pure tone of some intensity, whose frequency may be varied at will, is not easily obtainable. The results here described were obtained by the authors in the course of their work as technical officers of the Munitions Inventions Department, and are published by permission of the Department. The experiments were carried out at the Acoustical Research Section of the Department with the help of Major W. S. Tucker, R.E. The authors' best thanks are due to Major Tucker and his staff for their assistance.

The Seebeck siren, as described in books on sound, consists essentially of a revolving disc bored with holes, against which a stream of air is driven from a delivery pipe. The holes are equally spaced round a circle, and the pipe is fixed so that as the disc revolves the holes pass the mouth of the pipe in succession; when no hole is opposite the pipe the flow of air is partially or completely interrupted, and as a hole approaches and passes the flow of air waxes and wanes. When the disc is rotated sufficiently fast, a musical note is produced, whose frequency is equal to the number of holes passing the pipe per second.

In general, however, the note thus generated is not a pure tone, for the disturbance in the flow of air, though periodic, is not exactly simple harmonic. If the disturbance is represented by a Fourier's series, the series will contain more than one term, and the later terms must give rise to tones which are harmonics of the fundamental tone. Now a siren is a convenient source of sound when a note is required whose frequency may be varied at will, but in such cases the purity of tone may also be important. It is therefore of some importance to examine the relative intensities of the fundamental tone and its harmonics in ordinary cases. The present paper briefly investigates this matter. A theory is constructed which allows an approximate calculation of the relative intensities to be made in any given case. These calculations were carried out for an ordinary siren with circular holes and a pipe with an equal circular mouth, giving a strong first harmonic, and the results were compared with an experimental determination. The approximate truth of the theory was confirmed by the agreement of the observed and calculated ratios.

The theory, if strictly true, showed how a siren could be designed to give a pure tone by suitably modifying the shapes of the mouth of the pipe and the holes in the siren disc. Such a siren was made and tested, and found satisfactory. Though the first harmonic was still perceptible, its intensity was markedly reduced and the intensity of the higher harmonics was quite small. Of the total energy leaving the siren in the form of regular sound waves, about 95 per cent. appeared, in one set of experiments, to be possessed by the fundamental tone in the case of the new siren, for fundamental frequencies of about 350. The *theoretical* maximum percentage possible for any siren with circular holes is 82 per cent. Even for frequencies as low as 100 the octave in the new siren was seven times as weak as in the Seebeck siren used in the experiments.

We describe the theory, and our scanty experimental tests of it, in §§ 2–4, and then describe the construction of our pure tone siren in § 5. The required form of the holes in the siren plate is shown in fig. 5, and its purity of tone in figs. 3 and 6; fig. 3 providing a direct comparison with the Seebeck siren.

Since these results were obtained (1918), Major Tucker has done further experiments with the new siren, under somewhat different conditions. He has kindly provided us with the curves of fig. 7, which confirm the superiority of the new siren. His results for the Seebeck siren, however, do not seem to fit in with our theory of siren harmonics. This discordance may be only apparent, but much more exhaustive experiments are required than wartime conditions allowed before the proper value of this theory can be determined.

§ 2. *General Theory of Siren Harmonics.*—The sound waves originate from the region of the orifice of the air supply pipe, which at appreciable distances may be considered as a point source. The flux of air from this source is variable with the time; it may be regarded as a steady stream with superimposed oscillations. It is with these oscillations that we have to deal. The siren mechanism, in effect, is merely an apparatus for varying the total area of the orifice of the supply pipe, and it seems plausible to assume that *the actual flux of air at any moment is proportional to the area of the orifice exposed.* This must be very nearly true, whether the pipe abuts closely on the disc, or whether there is appreciable clearance between the pipe and disc, assuming a supply of air at constant pressure.

Waiving the special geometrical conditions of the problem, we may liken the siren to an isolated source in three dimensions, giving rise to a diverging series of waves, with a velocity potential, ϕ, given by

$$\phi = \frac{f(t-r/c)}{4\pi r},$$

where t is the time, r the distance from the source, and c the velocity of sound. The actual flux at the source is (omitting a density factor)

$$\underset{r\to 0}{\mathrm{Lt}}\left(-4\pi r^2\frac{\partial\phi}{\partial r}\right) = f(t).$$

Thus, on our own assumption, the function $f(t)$ must be taken to be proportional to the area of orifice exposed at time t.

Let this function be expressed in (say) a cosine series

$$f(t) = \mathrm{A}_0 + \mathrm{A}_1\cos 2\pi pt + \ldots + \mathrm{A}_n\cos 2\pi pnt + \ldots,$$

where p is the fundamental frequency, obtained from a revolution counter on the siren. Now, the radial velocity of the air at distance r at time t is given by

$$-\frac{\partial\phi}{\partial r} = \frac{f(t-r/c)}{4\pi r^2} + \frac{f'(t-r/c)}{4\pi rc}.$$

For values of r, large compared to the wave-length of the fundamental, the first of these terms may be neglected; on substituting the cosine series, the velocity is found to be

$$-\frac{p}{2rc}\Sigma n\mathrm{A}_n\sin 2\pi pn(t-r/c),$$

and the energy associated with the nth harmonic (*i.e.*, the intensity of the nth harmonic) is seen to be proportional to $n^2\mathrm{A}_n^2$. This result must apply to the siren source.

§ 3. *Harmonics of the Circular-hole Siren.*—As a typical example of ordinary siren apparatus, we will apply the foregoing to the case of a series of equally spaced *circular* holes and an air-delivery pipe, also circular in section and of the same diameter as the holes. For convenience we will neglect the curvature of the circle along which the holes are placed and suppose them to be in a straight line. We must first determine the Fourier series, expressing the area common to one of the holes and a section of the pipe in any position. The lower half of fig. 1 shows the graph of this area plotted against the position of the centre of the pipe; it is at once apparent that some at least of the early harmonics must be appreciable, from the notable departure of the graph from a sine curve.

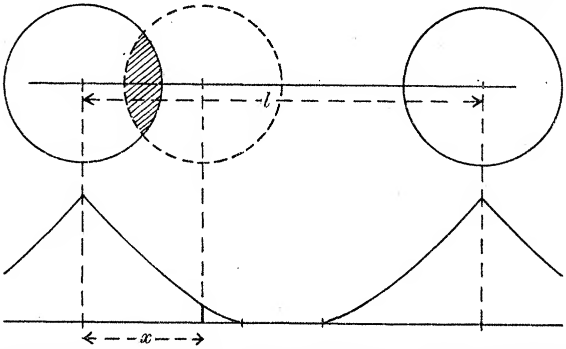

Fig. 1.—Siren holes and overlapping orifice, with the corresponding graph of A(x).

Let a be the diameter of a hole, l the distance between the centres of adjacent holes. We shall suppose first that $l \geq 2a$, so that the orifice never covers portions of two holes simultaneously. Let A (x) be the area common to the pipe and a hole, when the centre of the pipe is displaced a distance x from the centre of some particular hole. Clearly A (x) is an even function of x, with period l; thus

$$\mathrm{A}(x) = \mathrm{A}(-x) = \mathrm{A}(l-x), \tag{1}$$

and further

$$\mathrm{A}(x) = 0, \qquad (a \leq x \leq l-a). \tag{1'}$$

The function A (x) may be expanded in a Fourier's cosine series in the form

$$\mathrm{A}(x) = \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos 2\pi n x/l, \tag{2}$$

where the coefficients will be given by

$$a_n = \frac{2}{l}\int_0^l \mathrm{A}(x) \cos\frac{2\pi n x}{l}\,dx$$

$$= \frac{4}{l}\int_0^a \mathrm{A}(x) \cos\frac{2\pi n x}{l}\,dx, \tag{3}$$

in virtue of (1) and (1′). Integrating (3) by parts, and noting that $A(x) = 0$ when $x = a$, we have for $n \geq 1$

$$a_n = -\frac{2}{\pi n}\int_0^a \sin\frac{2\pi n x}{l}\frac{d\,A(x)}{dx}\,dx.$$

But it is easily seen geometrically that

$$d\,A(x) = -2\left(\tfrac{1}{4}a^2 - \tfrac{1}{4}x^2\right)^{\frac{1}{2}}dx,$$

and hence

$$a_n = \frac{2}{\pi n}\int_0^a \sin\frac{2\pi n x}{l}\,(a^2 - x^2)^{\frac{1}{2}}\,dx,$$

$$= \frac{4a^3}{l}\,\frac{1}{z}\int_0^1 \sin zt\,(1 - t^2)^{\frac{1}{2}}\,dt, \qquad (4)$$

where $z = 2\pi n a/l$. (5)

The coefficients a_n have thus been shown to depend simply on the function $S(z)$, defined by the equation*

$$S(z) = \frac{1}{z}\int_0^1 \sin zt\,(1 - t^2)^{\frac{1}{2}}\,dt. \qquad (6)$$

If we expand the sine and integrate term by term, we obtain, after reduction,

$$S(z) = \frac{\pi}{8}\sum_0^\infty \frac{(-)^n\left(\frac{1}{2}z\right)^{2n}}{\Gamma\left(n+\frac{3}{2}\right)\Gamma\left(n+\frac{5}{2}\right)}; \qquad (7)$$

this series, when written out in full, takes the form

$$S(z) = \frac{1}{1^2 . 3} - \frac{z^2}{1^2 . 3^2 . 5} + \frac{z^4}{1^2 . 3^2 . 5^2 . 7}, \qquad (8)$$

which converges for all values of z, and may be used for computation if z is not too large.

When $|z|$ is large, we can obtain an asymptotic expansion for $S(z)$ by constructing contour integrals of Barnes' type† for the series (7), and transforming these in the usual way. We can thus obtain, when $|\text{am}\,z| < \pi$,

$$S(z) - \frac{\pi}{2z^2}\,Y^1(z) \sim \frac{1}{z^2} + \frac{1}{z^4} - \frac{1^2 . 3}{z^6} + \frac{1^2 . 3^2 . 5}{z^8} - , \qquad (9)$$

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* This function is closely allied to Bessel's functions of order unity. It is a particular case of Lommel's function (Nielsen, 'Handbuch der Zylinderfunctionen,' p. 54), denoted in Nielsen's notation by

$$\frac{3^2}{\pi z^2}\,Z^1(z).$$

The properties of the function contained in equations (7), (8) and (9), which we require here, can be deduced from formulæ given by Nielsen, *loc. cit.*, pp. 54 and 228, or more simply obtained directly as indicated in the text.

† Whittaker and Watson, 'Modern Analysis,' 2nd ed., pp. 280, 337.

where $Y^1(z)$ is Neumann's second solution of Bessel's equation of order unity, as defined by Nielsen.[*] The asymptotic form of $Y^1(z)$ is well known,[†] and equation (9) is suitable for numerical work when $|z| \geq 8$. The complete expansion is

$$S(z) \sim \frac{1}{z^2}+\frac{1}{z^4}-\frac{1^2 . 3}{z^6}+\frac{1^2 . 3^2 . 5}{z^8}-\dots$$

$$+\frac{1}{z^2}\left(\frac{\pi}{2z}\right)^{\frac{1}{2}}\left[\cos\left(z-\tfrac{1}{4}\pi\right)\left\{-1-\frac{1^2 . 3 . 5}{(8z)^2\, 2\,!}+\frac{1^2 . 3^2 . 5^2 . 7 . 9}{(8z)^4\, 4\,!}-\dots\right\}\right.$$

$$\left.+\sin\left(z-\tfrac{1}{4}\pi\right)\left\{\frac{1 . 3}{(8z)\, 1\,!}-\frac{1^2 . 3^2 . 5 . 7}{(8z)^3\, 3\,!}+\dots\right\}\right].$$

The following short Table has been calculated by means of (8) and (9); fig. 2 exhibits the variation of S (z) as a graph.

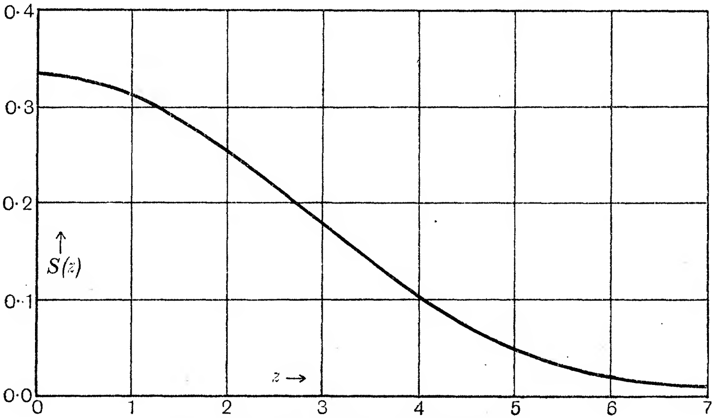

Fig. 2.—Graph of the function S (z).

Table I.

z.	S (z).	z.	S (z).
0	0·3333	4·5	0·0749
0·5	0·3278	5	0·0508
1	0·3117	6	0·0209
1·5	0·2864	7	0·0111
2	0·2540	8	0·0120
2·5	0·2169	9	0·01451
3	0·1780	10	0·01401
3·5	0·1400	12	0·00647
4	0·1050		

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[*] *Loc. cit.*, p. 11.

[†] Nielsen, *loc. cit.*, p. 156. Tables, from which the values of $Y^1(z)$ may be obtained, are given in 'Rep. Brit. Ass.,' 1913, p. 115.

If $\frac{1}{2}l \leq a \leq l$, so that in certain positions the orifice covers portions of two holes simultaneously, the total area covered is seen to be

$$\begin{aligned} &\mathrm{A}(x), && (0 \leq x \leq l-a); \\ &\mathrm{A}(x)+\mathrm{A}(l-x), && (l-a \leq x \leq \tfrac{1}{2}l; \end{aligned}$$

where $\mathrm{A}(x)$ is as above. The coefficients a_n are now given by

$$\begin{aligned} a_n &= \frac{4}{l}\left(\int_0^{l-a} \mathrm{A}(x) + \int_{l-a}^{\frac{1}{2}l} \{\mathrm{A}(x)+\mathrm{A}(l-x)\}\right) \cos\frac{2\pi n x}{l}\,dx \\ &= \frac{4}{l}\int_0^a \mathrm{A}(x) \cos\frac{2\pi n x}{l}\,dx, \end{aligned}$$

which is the same as before. Thus, combining the two cases, we see (on evaluating a_0) that for $o \leq a \leq l$ the effective area of the orifice is represented by the series

$$\frac{4a^3}{l}\left[\frac{1}{6} + \sum_{n=1}^{\infty} \mathrm{S}\left(\frac{2\pi n a}{l}\right) \cos\frac{2\pi n x}{l}\right], \tag{10}$$

and the intensities of the harmonics are proportional to

$$\{n\,\mathrm{S}(2\pi n a/l)\}^2. \tag{11}$$

The following Table shows the ratios of the intensities of the early harmonics for various values of a/l. To afford some indication of the applicability of this Table, it may be mentioned that the siren used in the tests of § 4 was provided with three alternative rings of holes comprising 4, 8, 16 holes respectively, the rings having radii of 4, 5, 6 inches respectively and the holes being $\frac{1}{2}$ inch in diameter. For a set of q holes spaced round a circle of radius r, $l = 2\pi r/q$ and $2\pi a/l = aq/r$; the values of this ratio for the three sets of holes mentioned are $\frac{1}{2}$, $\frac{4}{5}$, $\frac{4}{3}$ respectively, corresponding roughly to the first three rows in the Table.

Table II.—Relative Intensities of Harmonics for Circular-hole Sirens (Calculated).

$2\pi a/l$.	a/l.	Fundamental (n = 1).	Octave (n = 2).	Twelfth (n = 3).	3rd harmonic (n = 4).	4th harmonic (n = 5).
0·5	0·08	1·00	3·62	6·88	9·62	10·9
1	0·16	1·00	2·66	2·94	1·82	0·66
1·5	0·24	1·00	1·55	0·62	0·086	0·040
2	0·32	1·00	0·68	0·067	0·036	0·076
3	0·48	1·00	0·055	0·060	0·021	—
4	0·64	1·00	0·052	0·034	—	—
5	0·80	1·00	0·304	—	—	—
6	0·96	1·00	0·384	—	—	—

It is at once apparent that for small values of a/l only a small fraction of the total energy is concentrated in the fundamental, which is indeed considerably less intense than the harmonics immediately succeeding it. It appears further (the same thing is suggested by mere inspection of the curve of $S(z)$) that as a/l increases the relative prominence of the fundamental increases, and attains a maximum when $2\pi a/l$ is about 3 or 4, or a/l about 0·5; at this maximum the first harmonic contains only about 5 per cent. of the energy of the fundamental. To compare the energy in the fundamental with the total energy, we may proceed as follows: Differentiate (10) to obtain the series for $dA(x)/dx$ and apply the well-known theorem concerning the sums of the squares of the coefficients in a Fourier series. We find, for $0 \leq a \leq \frac{1}{2}l$,

$$\frac{l}{2}\cdot\left(\frac{8\pi a^3}{l^2}\right)^2\sum_1^\infty n^2\left[S\left(\frac{2\pi n a}{l}\right)\right]^2 = \int_0^l\left(\frac{dA(x)}{dx}\right)^2 dx = 2\int_0^a (a^2-x^2)\,dx = 4a^3/3,$$

or, putting $2\pi a/l = y$,

$$\frac{\pi}{3y^3} = \sum_1^\infty n^2[S(ny)]^2 \qquad (0<y\leq\pi), \tag{12}$$

so that the fraction of the energy concentrated in the fundamental is

$$\frac{[S(y)]^2}{\sum_1^\infty [nS(ny)]^2} = \frac{3y^3}{\pi}[S(y)]^2.$$

The following Table shows the value of this fraction for various values of y or $2\pi a/l$:—

Table III.

$2\pi a/l$.	$\frac{1}{2}$.	1.	$1\frac{1}{2}$.	2.	3.
Fraction	0·013	0·093	0·265	0·493	0·818

Thus for $2\pi a/l = 3$, or $a/l = 0·48$, 82 per cent. of the energy occurs in the fundamental, and the tone is fairly pure. For the extended range $\frac{1}{2}l < a \leq l$, the expression for the sum of the series in (11) is somewhat complicated, and we shall not pause to evaluate it. It seems probable, however, that the best approximation to a pure tone is obtained for values of a/l in the neighbourhood of 0·5.

§ 4. *Experimental Verification.*—It must be remembered that aural estimates of the relative intensities of sounds do not agree with the calculated intensities; the "loudness" of a note is more nearly proportional to the amplitude of an oscillation than to its square. The actual intrusion of the octave, twelfth, etc., will, therefore, not be so pronounced as the Table would appear to show; but it is not difficult to detect them with the naked ear.

A series of numerical tests were carried out in conjunction with Major Tucker, using his hot-wire microphone.* The microphone was mounted in the orifice of a Helmholtz resonator placed near the siren, and connected with an amplifier, rectifier, and galvanometer. The evidence available at the time of these experiments showed that the galvanometer deflection was proportional to the *energy* of the resonant vibrations of the air in the resonator. The resonator has a single free period (in general) and does not respond to the octave of its own particular note. The siren was run at a series of speeds, and the deflection read for each speed. If the siren note is pure, the curve of deflection against speed should show a single peak, the abscissa of which must correspond to the note of the resonator; the presence of other peaks indicates that the frequency corresponding to the resonator is a constituent of the siren note for other speeds of rotation.

The dotted curve in fig. 3 is such a curve, though on this occasion the Helmholtz resonator (owing to a special orifice) had two free periods, of frequencies 82 and 93.† It will be seen at once that tones of these frequencies were constituents of the siren note when the speed was such that the frequencies of the fundamental were 41 and 47.

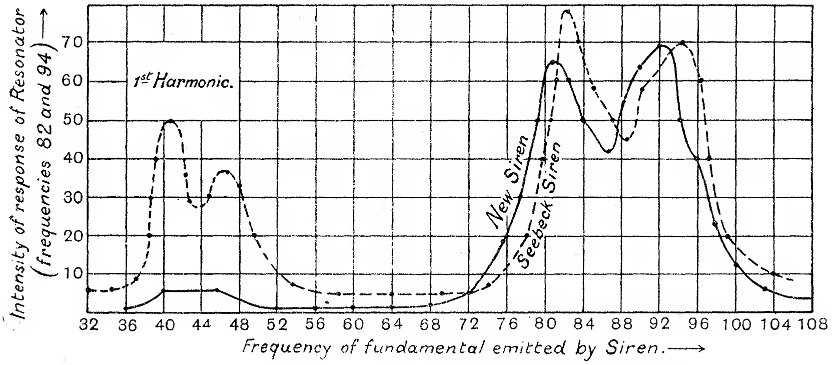

FIG. 3.—Comparative test of purity of notes emitted by Seebeck siren (dotted) and pure tone siren (full line). The tests were carried out on different days with different temperatures, hence the shift in the positions of the maxima.

The ratios of the ordinates for these pairs of frequencies (after reducing their zero to the general level of the curve) are 45/74 and 32/65, or 0·61 and 0·49, mean 0·55. Before we compare this result with the calculated values of Table II, we must observe that the ratio thus measured is the ratio, not of the octave to the fundamental in a given note, but the ratio of a given tone, when it is the fundamental, to the same tone when it is the octave of the fundamental. If siren holes, q in number, are set along a circle of radius, r, and if the siren is making R revolutions per second, then $2\pi a/l = aq/r$, $x = 2\pi \mathrm{R}rt$, and the harmonic $\mathrm{S}(2\pi na/l)\cos 2\pi nx/l$ becomes

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* We understand from Major Tucker that he hopes shortly to be allowed to publish a full account of his hot-wire microphone, which is at present confidential. Reference must be made to this paper for further details of the apparatus.

† Frequencies as low as these were employed because the tests were really only incidental experiments in a course of work, the application of which was entirely to low (aeroplane) notes.

$$\mathrm{S}\,(naq/r)\cos 2\,\pi n\,\mathrm{R}qt.$$

In a test with a resonator, we compare the intensity of a certain tone when it is the fundamental ($n = 1$, $\mathrm{R} = \mathrm{R}_1$) with that of the same tone when it is the nth harmonic of the fundamental obtained by running the siren n times as slowly ($n = n$, $\mathrm{R} = \mathrm{R}_1/n$). The product $n\mathrm{R}$ has the same value in the two cases, and the ratio of the intensities is

$$\left(\frac{\mathrm{S}\,(naq/r)}{\mathrm{S}\,(aq/r)}\right)^2,$$

which differs from (11) in the absence of the n^2 factor. It is important not to overlook this point in analysing siren tests.

For the test in question, $q = 16$, $r = 6$. The diameter of the holes was $\frac{1}{2}$ inch but the air-supply tube was slightly smaller, $\frac{2}{5}$ inch. When the tube and holes have not the same diameter, the calculations of § 3 become much more elaborate, but it can be shown that if the diameters differ only slightly, a first approximation is obtained by taking for a the mean of the diameters. Thus in our case $a = 0{\cdot}45$, and so $aq/r = 1{\cdot}2$. The required ratio,

$$\left(\frac{\mathrm{S}\,(2aq/r)}{\mathrm{S}\,(aq/r)}\right)^2$$

is one quarter of the corresponding entry in Table II, and so is found to be $\frac{1}{4}(2{\cdot}22)$ or 0·55. This lies satisfactorily between the observed values. (In practice, the measured ordinates of the peaks are, unfortunately, not so sharply defined as the measured abscissæ.)

This agreement provides some verification of our theory, though much more exhaustive tests would be desirable. It is sufficient, however, to justify a use of the theory with some confidence in an attempt to design a siren to give a pure tone.

§ 5. *Construction of a Siren to give a Pure Tone.*—Accepting the general theory of a siren given above, to obtain a pure tone we should attempt to modify the shape of the holes, or of the cross-section of the supply tube, or both, so that the effective area of the orifice (the area common to tube and hole) is proportional to the sine of the displacement. It is clear at once that this cannot be attained so long as the tube and holes have the same shape; for the maxima of the curve of effective area, which will occur when the tube exactly covers a hole, will always be sharp, as in fig. 1, the differential coefficient here changing suddenly from a non-zero positive value to a negative value.

If the orifice were reduced to a narrow straight slit,* and the boundary of the cut-out portion of the disc be itself a sine-curve (or double sine-curve), the object would be achieved. The use of a narrow slit for the orifice is, however, open to objections; there is difficulty in getting sufficient intensity unless the slit is made very long; further, there are mechanical difficulties in cutting a continuously sinusoidal hole, owing to the severing of the material, so that it can only be cut in the edge of the plate.

If, however, the orifice is altered merely to a *rectangular* section, the following construction (which is believed to be novel) may be adopted. Let a rectangular aperture of breadth a, and sufficient height to overlap all the ordinates concerned, pass over the region bounded between the curve $y = f(x)$ and the x-axis; and suppose that $f(x) = 0$ for $-a \leq x \leq 0$. Our requirement is then that

$$\int_{x-a}^{x} f(x)\,dx \propto 1-\cos kx$$

for $x \geq 0$. Differentiating we have

$$f(x)-f(x-a) \propto \sin kx.$$

Omitting the constant of proportionality, we can now build up the following system of values of $f(x)$:—

$$\begin{aligned} f(x) &= 0 && (-a \leq x \leq 0), \\ f(x) &= \sin kx && (0 \leq x \leq a), \\ f(x) &= \sin kx + \sin k(x-a) && (a \leq x \leq 2a), \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot && \\ f(x) &= \sin kx + \ldots + \sin k(x-\overline{p-1}a) && [(p-1)a \leq x \leq pa]. \end{aligned}$$

The period in x is $2\pi/k$; putting $2\pi k = na$, and summing the expression for $f(x)$, we find

$$f(x) = \frac{\sin p\pi/n}{\sin \pi/n} \sin \frac{2\pi}{n}\left(\frac{x}{a}-\frac{p-1}{2}\right) \qquad (p-1 \leq x/a \leq p).$$

By the nature of the case, $f(x)$ must never be negative, and must be periodic in na; it follows that n must be an integer, and that the last equation must hold for values of p up to $n-1$; also, $f(x) = 0$ for $x/a = p = n-1$, and $f(x)$ must continue to be zero throughout the nth section. Examples of the complete graph of $f(x)$, for the two cases of n even and n odd, are shown in fig. 4 ($n = 6$, $n = 5$). The curious shape of the curve for n odd near its mid point is of some interest.

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* This has been suggested previously; see Koenig, 'Ann. der Phys. und Chem.,' 2nd series, vol. 14, p. 369 (1881). In this siren the edge of the disc had the form of a sine-curve; Koenig gives no evidence as to the real purity of the tones obtained.

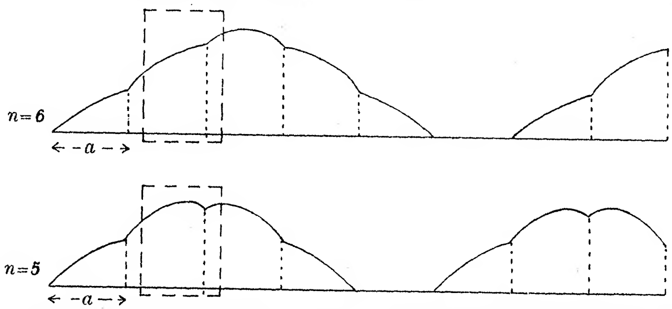

FIG. 4.—Calculated forms of holes for pure tone siren, with rectangular orifice (orifice shown dotted).

A siren disc of this type was constructed in Major Tucker's workshop; the disc was bored with twelve holes of the shape of the first diagram in

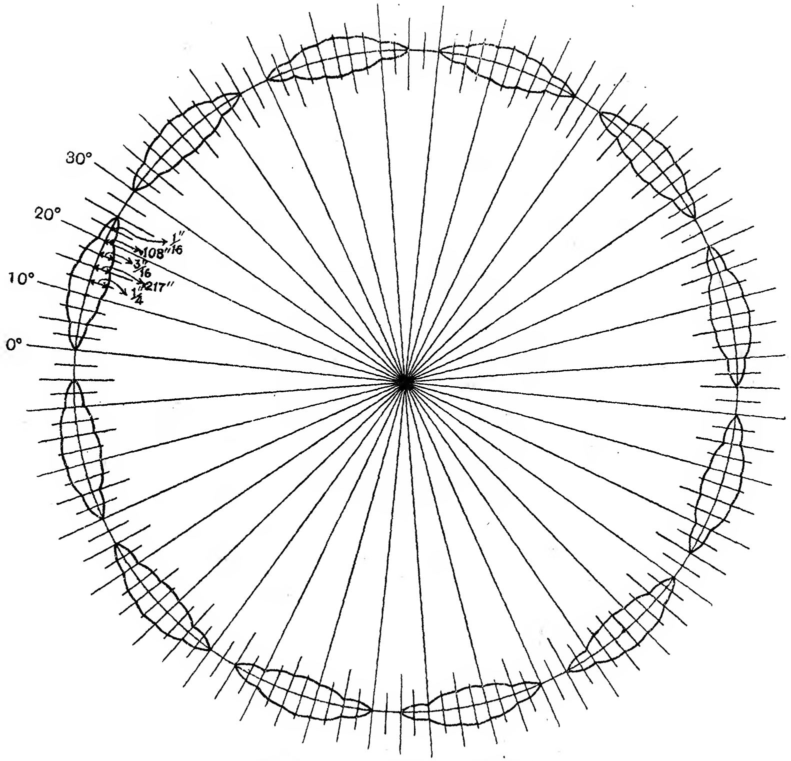

FIG. 5.—Disc of pure tone siren.

fig. 4, set round a circle of radius 4·3 inches, the ordinates being marked off radially* on each side of the base circle; a drawing of the complete disc is shown in fig. 5.

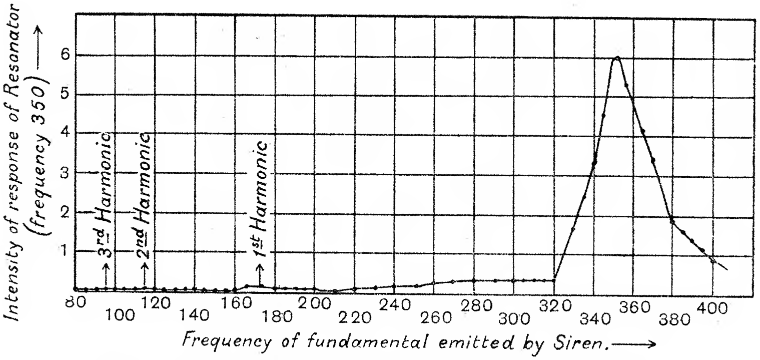

Fig. 6.—Test of purity of siren note (pure tone siren).

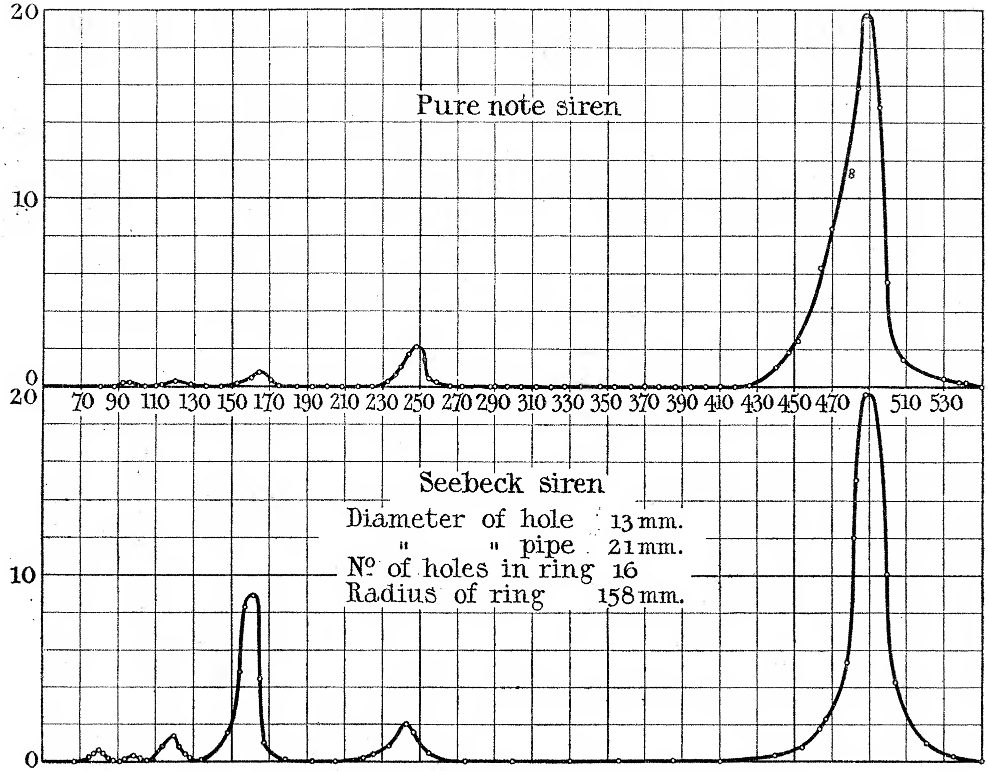

Fig. 7.—Comparative test of purity of notes, frequency 488.

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[*] The analysis is easily modified to apply to the case of the circular base-line, but the actual corrections are small. The orifice is, of course, a section bounded by arcs and radii.

The results of tests of this siren with resonators are shown in figs. 3, 6, 7, for fundamentals of frequencies 80–90, 350, 488.

It will be seen that there is a marked improvement in the purity of the tone. The purity appears to be greater for the high frequencies than for the low ones; in fig. 3 the peak corresponding to the octave is about 7.5 per cent. of that corresponding to the fundamental (frequency 80 to 100); in fig. 4 (frequency 350) the fraction has diminished to 1 or 2 per cent. (It must be remembered, in accordance with § 4, that the ratios of the intensities of octave to fundamental in the actual notes must be four times these, namely, 30 per cent. and 4–8 per cent.) It would, perhaps, be expected that irregular disturbances and imperfections, leading to higher values for the later terms in the Fourier series, would be more prominent for low speeds than for high speeds. Even for the low speed, however, the first harmonic is about seven times as intense in the Seebeck siren used as in the new siren.

§ 6. *Summary.*—The ordinary siren can be regarded as a point source of air of variable flux, the flux being proportional to the area of the orifice exposed by the holes in the disc; in general, the tone from such a siren is far from pure. The relative intensities of the harmonics for a siren with circular holes and a circular orifice are calculated and compared with experiment; it is concluded that a fairly pure note should be obtained from a siren of this type in which the distance between the centres of adjacent holes is twice the diameter of the holes. If, however, the orifice is rectangular in section, the holes can be so shaped that the area of the orifice exposed varies exactly as the size of the displacement, such a siren should produce a very pure tone. Experimental tests of a siren constructed on these lines are quoted and discussed.